Quantum Correlations and Quantum-Classical Transition of a Cavity Field

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• Introduction – A bit of History
  - EIT in Optical Cavity
  - Piza’s work: classical field with a few photons

• Interaction of Two atoms with a Cavity mode
  - Atom-field Interaction Model
  - Generation of Correlations – asymptotic regime
    - Coherent Driving
    - Incoherent Driving
  - Time to get the steady state

• Perspectives
Electromagnetically Induced Transparency (EIT):
Introduction – EIT in Optical Cavity with few atoms

Behaviour close to Resonance with a control field

$$\omega_p$$

$$\frac{\omega_p - \omega}{\gamma}$$

Absorption

$${\Omega_c}$$

$${\Omega_p}$$

Rev. Mod. Phys., Vol. 77, No. 2, April 2005
Introduction – EIT in Optical Cavity with few atoms

V = 17 m/s !!!


V = 17 m/sec

Cycling at the speed of light?
Introduction – EIT in Optical Cavity with few atoms

\[ C = \frac{g^2}{2\kappa\gamma} \approx 1 \]

- \((g, \kappa, \gamma) = 2\pi \times (\leq 6, 3, 3)\text{MHz}\)
- Finess: \(\omega/\kappa = 60000\)
- Length: 500\(\mu\text{m}\)
Introduction – EIT in Optical Cavity with few atoms

Transmissão%\[
\begin{align*}
&\text{probe laser} \\
&\text{atom} \\
&\text{cavity}
\end{align*}
\]

--- Cavidade vazia
--- Átomo de 2 níveis

Transmissão%
EIT em cavidades

Transmissão (%)

Cavidade vazia

Átomo de 2 níveis

Campo de controle

Transmissão (%)

- Cavidade vazia
- Átomo de 2 níveis
- Campo de controle

\( \Delta P/\kappa \)
Introduction – EIT in Optical Cavity with few atoms

M. Mücke et al, Nature 465, 755 (Junho 2010)
**EIT Conditions** - $\Omega_c \gg \Omega_p \rightarrow P1 \approx 100\%$ - $P2 \approx P3 \approx 0$

Quantum Model = Semi-Classical Model
(time-dependent Amplitude of the cavity field)
1- EIT - It can be described by two coupled CLASSICAL oscillators:

Classical analog of electromagnetically induced transparency

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Quantum field EIT?
2- A very weak field can be classical

Single two-level atom coupled to a driven cavity mode with strong dissipation $\rightarrow$ classical behaviour of the field!

Quantum-To-Classical Transition in Cavity Quantum Electrodynamics

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PRL 105, 163601 (2010)
Quantum-To-Classical Transition in Cavity Quantum Electrodynamics

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$V_{\pm,1} \leftrightarrow V_{\pm,2}$
Let’s change the Meter:

1 atom $\rightarrow$ 2 atoms
Measuring properties of a cavity field – ONE atom

- When \( \langle n \rangle = 1.0 \)
- When \( \langle n \rangle = 2.0 \)
- When \( \langle n \rangle_{\text{max}} = 50.0 \)
- \( g = 5.0 \kappa \)
Measuring properties of a cavity field – TWO atoms

\[ <n>_{\text{max}} = 0.1 \]
\[ <n>_{\text{max}} = 1.0 \quad a = 10\kappa \]
\[ <n>_{\text{max}} = 2 \quad g = 10\kappa \]
\[ <n>_{\text{max}} = 20 \quad a = 10\kappa \]
\[ <n>_{\text{max}} = 20 \quad g = 5\kappa \]
BUT: Intense cavity field is still made of particles (photons!)

Brownian Motion:

Signature of the particle nature a fluid!

Can we see the quantum nature of the field even in the macroscopic regime?
1- Semi-Classical Model:

\[ S_+ = S_+^\dagger = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{ikx_j} \sigma_+^j, \sigma_+^j = |e\rangle_j \langle g| \]

\[ H_I = \left[ \Omega(t) \sqrt{N} e^{i\omega t} S_+ + h.c. \right] \]

\[ \rho(0) = \rho_1 \otimes \rho_2 \quad \rightarrow \quad \rho_1(t) \otimes \rho_2(t) \]

CLASSICAL FIELD \(\rightarrow\) NO CORRELATIONS BETWEEN THE ATOMS
1- Quantum Model: \[ H_I = g \sqrt{N} (aS_+ + H.c.) \]

Effective Master Equation – Atoms:

\[ \dot{\rho}_a = -i[H_{SC}, \rho_a] + \frac{g^2N}{\kappa} \mathcal{L}[S_-] \rho_a \]

Classical Field Dynamics

Effective Interaction between the atoms Induced by the quantum mode

Rossatto et al, PRL 107, 153601 (2011)
Quantum Field \times \text{Classical Field}

Atom 1 \quad \text{Atom 2}

1 \text{ Photon}

\begin{align*}
\text{Classical Field} & \rightarrow \text{Can exchange any amount of energy.} \\
\text{Quantum Field} & \rightarrow \text{Can exchange only 1 photon/atom.}
\end{align*}

Rossatto et al, PRL 107, 153601 (2011)
1- Quantum Field: \[ H_I = g \sqrt{N} (aS_+ + H.c.) \]

A– Coherent Driving – Steady State

\[ |\Psi(0)\rangle = |e, g\rangle \]

Rossatto et al, PRL 107, 153601 (2011)
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Coherent Driving: Particular Field?

Strong driving $\rightarrow$ coherent state!

\[ |\text{cavity}\rangle = |\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \]

Rossatto et al, PRL 107, 153601 (2011)
1- Quantum Field: \[ H_I = g \sqrt{N} (aS_+ + H.c.) \]

B– Thermal Field – Steady State

\[ g = 0.1 \kappa \]

Rossatto et al, PRL 107, 153601 (2011)
1- Quantum Field: 

\[ H_I = g \sqrt{N} \left( a S_+ + H.c. \right) \]

B– Thermal Field – Steady State

\[ \kappa \gg g \sqrt{N}, \quad t \gg 1/\kappa \]

Reduced Master Equation for the atoms:

\[ \dot{\rho}_a = \Gamma_{eff} \left( n_{th} + 1 \right) \mathcal{L} \left[ S_- \right] \rho_a + \Gamma_{eff} n_{th} \mathcal{L} \left[ S_+ \right] \rho_a \]

Rossatto et al, PRL 107, 153601 (2011)
1- Quantum Field: \( H_I = g \sqrt{N} (a S_+ + H.c.) \)

B– Thermal Field – Steady State

\( (n_{th} >> 1) \)

\( |g, g\rangle \rightarrow \rho^{ss}_a = \frac{1}{3} (|\Phi^+\rangle \langle \Phi^+| + |\Phi^-\rangle \langle \Phi^-| + |\Psi^+\rangle \langle \Psi^+|) \rightarrow QD = 1/3 \)

\( |e, g\rangle \rightarrow \rho^{ss}_a = \frac{1}{6} 1 + \frac{1}{3} |\Psi^-\rangle \langle \Psi^-| \rightarrow QD \approx 1/8 \)

Increasing \( n_{th} \) \( \Rightarrow \) THERE IS ALWAYS CORRELATIONS!!!

\( |\Psi^\pm\rangle = 1/\sqrt{2} (|e, g\rangle \pm |g, e\rangle) \)

\( |\Phi^\pm\rangle = 1/\sqrt{2} (|g, g\rangle \pm |e, e\rangle) \)

Rossatto et al, PRL 107, 153601 (2011)
Time to reach the steady state: Coherent Driving

Smaller $<n> \rightarrow$ faster the system reaches the steady state!

Rossatto et al, in preparation
Time to reach the steady state: Thermal Field

\[ g = 0.1\kappa \]

Increasing \( n_{th} \)

\[
\begin{align*}
\text{Quantum Discord} & \\
& \begin{cases}
\text{\( n_{th} = 0.1 \)} \\
\text{\( n_{th} = 0.5 \)} \\
\text{\( n_{th} = 1.0 \)} \\
\text{\( n_{th} = 2.0 \)}
\end{cases}
\end{align*}
\]

Rossatto et al, in preparation
Caracteristic Time to reach the steady state:

\[
\tau_c = \frac{1}{4 \left( \frac{g}{\kappa} \right)^2 \left[ 2n_{th} + 1 - \sqrt{n_{th} (n_{th} + 1)} \right]}.
\]

* Stronger \( g \) \( \rightarrow \) faster the system reaches steady state!

* Higher \( n_{th} \) \( \rightarrow \) faster the system reaches steady state!

Rossatto et al, in preparation
1- What happens with other driving fields? E.g. Parametric driving (squeezing!)

2- What happens when we increase the number of atoms inside the cavity?

3- Implementation: what is the best system to see this quantum nature a macroscopic cavity field?

4- What Happens in the strong coupling regime $(g \gg \kappa)$ ?
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